Visualising program dataflow with string diagrams

S-REPLS 13 / Fun in the Afternoon

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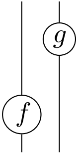
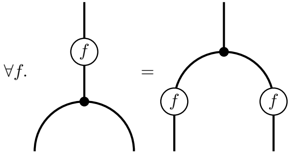
String diagrams

- String diagrams are a graphical notation for terms in different types of monoidal categories
  - The term $(f \otimes \mathsf{id}) \circ (\mathsf{id} \otimes g)$ is represented by the string diagram:



  - Equations of terms arising from the monoidal structure are captured by isotopy of string diagrams
- Cartesian monoidal categories (i.e. $\otimes = \times$ and $I = 1$) admit a natural copy-delete comonoid:



`sd-lang`

- Toy language for programs
- Syntax: essentially lambda calculus with operations and recursive let
  - `bind x = v1 y = v2 … in v`
  - values are variables, thunks, or operations `op(v1, v2, …)`
    - `plus(x, y)`, `eq(x, y)`, `if(cond, tb, fb)`, etc.
- Semantics: hierarchical hypergraphs
  - a model of string diagrams for symmetric monoidal categories with copy-delete
- (D. R. Ghica, Muroya, and Ambridge 2021)

Example: factorial

```
bind fact = lambda(x .
  if(eq(x, 0),
    1,
    times(x,
      app(fact,
        minus(x, 1)
      )
    )
  )
)
in app(fact, 5)
```

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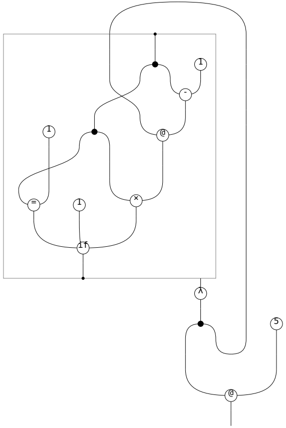

Figure 1: factorial as a string diagram

Representation of programs

Traditional representation: abstract syntax tree

```
bind("fact",
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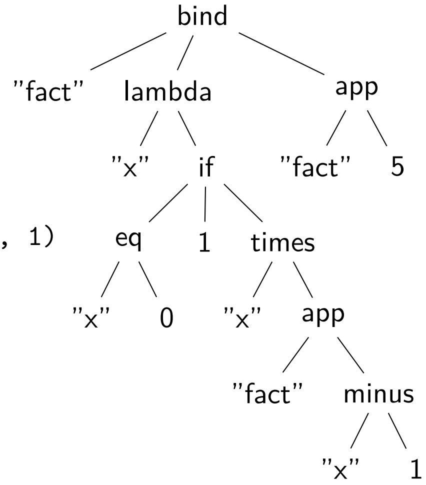

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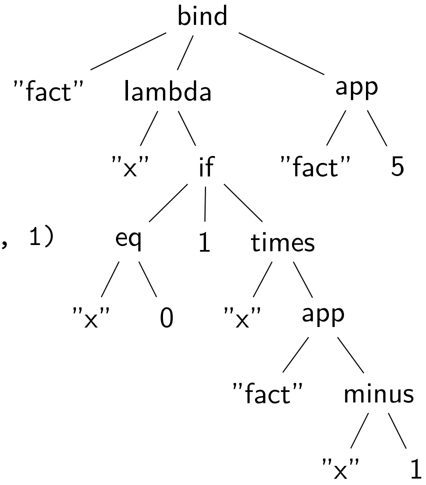


Compiler optimisations are described by semantic-preserving transformations on these ASTs given by rewrite rules.

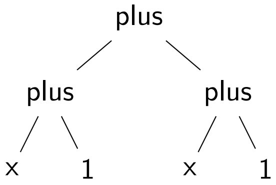
ASTs do not support sharing, or α-equivalence I

Consider the expression $(x+1)+(x+1)$ (where x is free).

This is represented by the `sd-lang` expression

```
plus(plus(x, 1), plus(x, 1))
```

Its AST is



ASTs do not support sharing, or α-equivalence II

Problem: The term obtained by the α-invariant substitution $[x \mapsto y]$ is represented by a different AST.

Consequence: The optimisation $\texttt{plus}(x_1, x_2) \to \texttt{times}(x_1, 2)$ needs to do a non-trivial computation to be valid, namely checking that $x_1 \equiv_\alpha x_2$.

- Can leverage de Bruijn indices, nominal techniques...

String diagrams do support sharing, and α-equivalence

Our string diagrams are equipped with a natural copy-delete comonoid.

This allows for a more meaningful representation of this program as the string diagram:

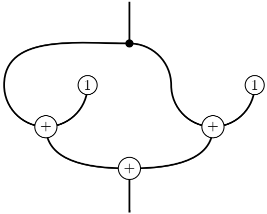


Figure 2: $(x+1)+(x+1)$ — observe that x does not appear in the diagram!

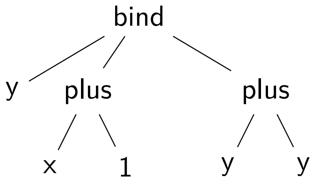
Nodes represent operations, and edges represent dataflow (e.g. of values)!

ASTs do not support binding and shadowing

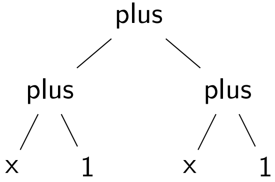
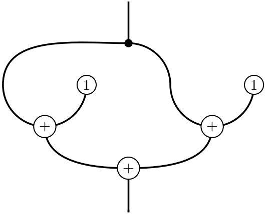
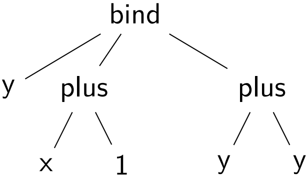
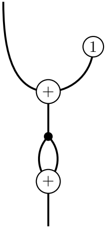
Another way to write this program:

```
bind y = plus(x, 1) in plus(y, y)
```

AST:

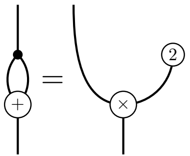


ASTs vs string diagrams

AST	String diagram
	
	

Compiler optimisations as string diagram rewriting

The optimisation we care about is



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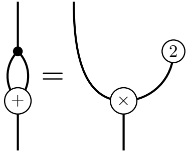
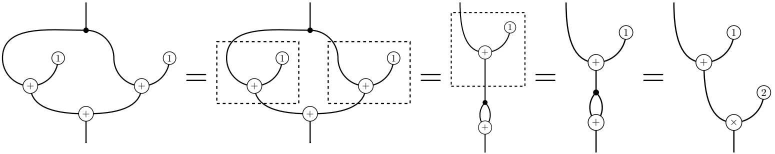


Derive



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  - ✗ (✓?) Not very well studied, **lack of tooling**(!)

How to draw a string diagram

- Hypergraphs quotient monoidal categories with copy-delete
- For each hypergraph, we need to pick a representative monoidal term
  - Involves (non-canonically) foliating the hypergraph into layers, and determining the order of operations (which determines how many ‘swaps’ are needed)
  - *Aesthetically-pleasing diagram heuristic*: minimise the number of layers, and the number of swaps (NP-hard)
- Given a monoidal term, we can construct a big LP to determine the coordinates of each node and positioning of edges (Tataru and Vicary 2023)

Demo

- Also available at https://sd-visualiser.github.io/sd-visualiser

Future work and references

- LLVM's Multi-Level Intermediate Representation (MLIR)

References

Ghica, Dan R., Koko Muroya, and Todd Waugh Ambridge. 2021. "A Robust Graph-Based Approach to Observational Equivalence." September 23, 2021. https://doi.org/10.48550/arXiv.1907.01257.

Ghica, Dan, and Fabio Zanasi. 2023. "String Diagrams for λ-Calculi and Functional Computation." October 19, 2023. https://doi.org/10.48550/arXiv.2305.18945.

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